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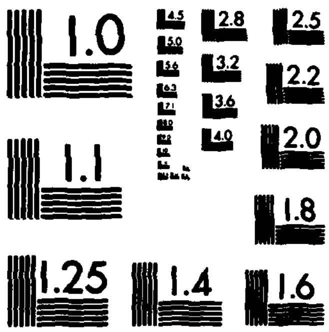

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USE OF SHEAR LAG FOR COMPOSITE MICROSTRESS ANALYSIS - RECTANGULAR ARRAY

S.B. BATDORF AND ROBERT W.C. KO

Sponsored by the
Department of the Navy
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Under Contract No. N00014-76-C-0445

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER UCLA-ENG-83-36	2. GOVT ACCESSION NO. AD-A134 165	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) USE OF SHEAR LAG FOR COMPOSITE MICROSTRESS ANALYSIS - RECTANGULAR ARRAY		5. TYPE OF REPORT & PERIOD COVERED INTERIM Jan.-Aug. 1983
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) S.B. Batdorf and Robert W.C. Ko		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0445
9. PERFORMING ORGANIZATION NAME AND ADDRESS School of Engineering and Applied Science UCLA, Los Angeles, CA 90024		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research 800 North Quincy St., Arlington, VA 22217 Attn: Dr. Y. Rajapakse		12. REPORT DATE July 1983
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Office of Naval Research - Branch Office 1030 East Green Street Pasadena, CA 91101		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Electric Analogue
Unidirectional Composites
Shear Lag
Mechanics of Failure
Micromechanics
Stress Distribution
Damage Accumulation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A number of authors have employed shear lag theory to find the stress distribution near cracks in a uniaxially reinforced composite. Practical application of these equations has been hampered by the fact that they include a parameter h/d the magnitude of which is unknown. This parameter is evaluated for composites in which the fibers forms a rectangular array.

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Forward

This paper was prepared for inclusion in the Symposium entitled "1983 Advances in Aerospace Structures and Materials," to be held during the Winter Annual Meeting of ASME, Boston, November 13-18, 1983.

USE OF SHEAR LAG FOR COMPOSITE MICROSTRESS ANALYSIS - RECTANGULAR ARRAY

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ABSTRACT

A number of authors have employed shear lag theory to find the stress distribution near cracks in a uniaxially reinforced composite. Practical application of these equations has been hampered by the fact that they include a parameter h/d the magnitude of which is unknown. This parameter is evaluated for composites in which the fibers form a rectangular array.

NOMENCLATURE

- d distance between rivet lines or distance between rod centers
- d_1 distance between next-to-nearest fiber pair
- d_2 distance between nearest fiber pair
- h plate or shell thickness
- $\frac{h}{d}_{c,h}$ $\frac{h}{d}$ of the composite in the horizontal direction
- $\frac{h}{d}_{c,v}$ $\frac{h}{d}$ of the composite in the vertical direction
- $\vec{j}$ current density in electrolyte
- j_n normal component of the current density
- r_o radius of a fiber or a rod
- r_1 distance from a negative line force
- r_2 distance from a positive line force
- w_n axial displacement of the n'th fiber
- z axial coordinate
- F_i force in the i'th stiffener or fiber in a composite
- G shear modulus of the matrix
- Gh effective shear stiffness of the composite
- I current generated by the power supply
- I_i current in the i'th rod
- α $\frac{d_1}{d_2}$
- ρ_e resistivity of the electrolyte
- τ shear stress in matrix
- ϕ electrical potential in rod or electrolyte

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INTRODUCTION

The analysis of failure of initially crack-free continuous-fiber composites is complicated because such failures are not governed by weakest link considerations. Instead they are a result of damage accumulation. To account for crack growth we must know the microstresses in the neighborhood of the crack tip. At present such understanding is limited to unidirectionally reinforced composites and, even there, is somewhat fragmentary.

Most attempts to find stress distributions near the crack tip are based on shear lag theory. This approach simplifies elastic theory by assuming that all displacements are parallel to the fibers and that the matrix between the fibers carries only shear stress. The fundamental equation of shear lag theory, originally developed for the stiffened plates and shells employed in aircraft, takes the form

$$\frac{dF_i}{dz} = Gh\,\frac{-w_{i-1} + 2w_i - w_{i+1}}{d} \tag{1}$$

By analogy, the corresponding equations for composites were assumed by Hedgepeth (1,2) and later by many others to take the same form in the case of a linear array, and in the case of a square array of fibers to take the form

$$\frac{dF_{ij}}{dz} = F'_{ij} =$$

$$Gh\,\frac{-w_{i,j-1} - w_{i-1,j} + 4w_{ij} - w_{i,j+1} - w_{i+1,j}}{d} \tag{2}$$

This would be correct for a square array if it had a very low fiber volume ratio and the matrix was replaced by thin plates as shown in Figure 1. But for high fiber volume ratio and/or conventional matrices it is quite unclear how to estimate h/d or whether the assumption that only nearest neighbors interact is a good one.

Some work has already been done on these questions. Batdorf (3) gave an exact solution for the interaction between two infinitely long rigid rods immersed in an infinite elastic matrix with displacements $\pm w_o$. The result in this case was that

$$\frac{dF_+}{dz} = -\frac{dF_-}{dz} = G\,\frac{h}{d}\cdot 2w_o \qquad (3)$$

where

$$\frac{h}{d} = -/\ln[(1 \pm \sqrt{1-(2r_o/d)^2})/(2r_o/d)] \qquad (4)$$

Here r_o is the rod radius and d is the distance between rod centers. It follows that h/d is a function of r_o/d varying from 0 for $r_o = 0$ to ∞ for $r_o = d/2$. Unfortunately the technique employed cannot be extended to more than two fibers. This is because whereas the displacement contours in the matrix surrounding two rods of circular cross-section are circular cylinders, for more than two rods they are not.

The interaction between nearest neighbors for the case of a square array was found by Batdorf and Ghaffarian (4) using an electric analogue (5). The present papers extends this approach to the determination of interaction between nearest neighbors in a rectangular array. To supplement the solution found experimentally employing the electric analogue, an analytical approach valid for very small fiber volume ratio is also presented. The two solutions are found to be in good agreement in the limiting case where the analytical solution is valid.

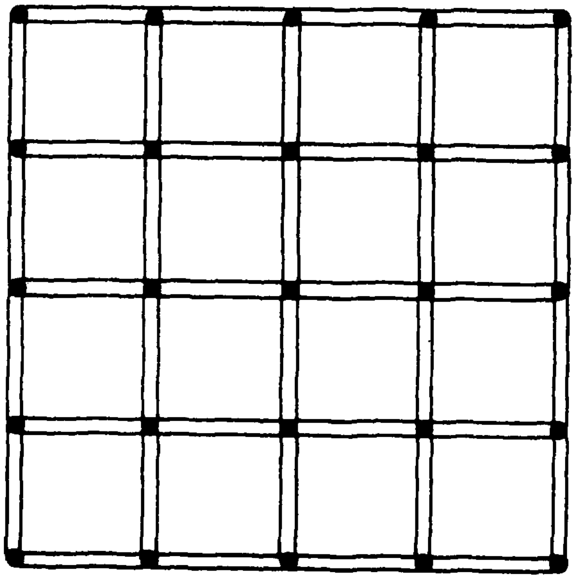

FIG. 1 HYPOTHETICAL MATERIAL OBEYING CONVENTIONAL SHEAR LAG EQUATIONS

Analytical Solution for Rectangular Array

For the case of a rectangular array, two parameters have to be taken into account at the same time; namely, [illegible] for the two nearest neighbors and [illegible] for the next-to-nearest neighbors. The analysis for such an array can be carried out by a consideration of the load transfer due to a set of equal but opposite forces directed to two nearest neighbors as in Figure 2 and to two next-to-nearest neighbors as in Figure 3. The analysis is valid in the limiting case of small r_o/d.

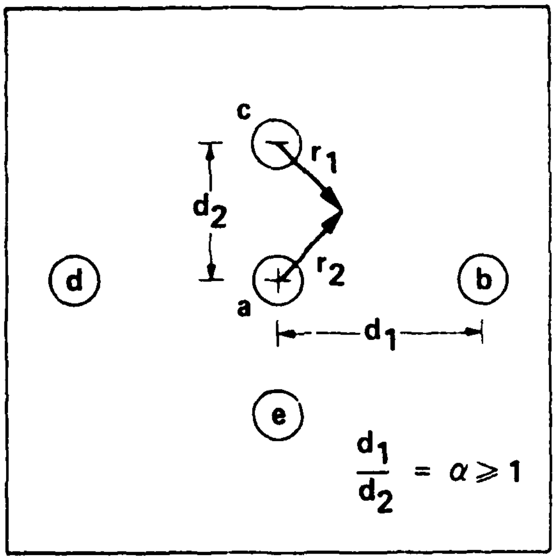

FIG. 2 LOADS APPLIED TO NEAREST NEIGHBOR PAIR - CASE A

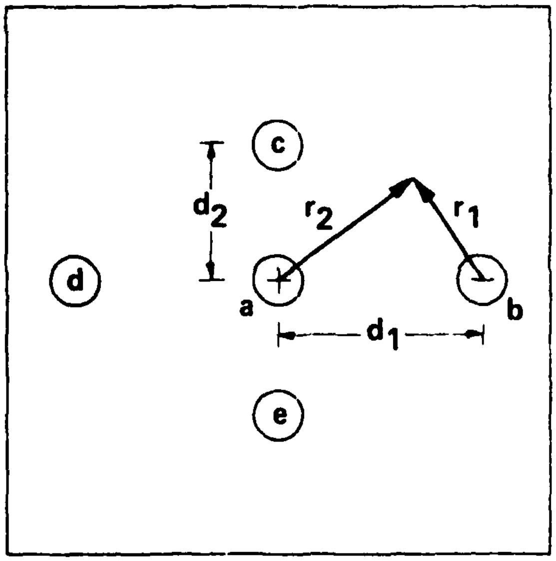

FIG. 3 LOADS APPLIED TO NEXT-TO-NEAREST NEIGHBOR PAIR - CASE B

In Case A, the shear lag equation becomes

$$F_a' = G[(\frac{h}{d})_{c,v}(2w_a - w_c - w_e) + (\frac{h}{d})_{c,h}(2w_a - w_b - w_d)] \qquad (5a)$$

$$= G\left(\frac{h}{d}\right)_{i,v} (w_a - w_c) \tag{5b}$$

where subscript c denotes the value for the 2-fiber interaction in a rectangular array composite, and i denotes the value for a pair of isolated fibers. For sufficiently small r_o/d_2,

$$w_a = -w_c = \frac{F'}{2\pi G} \ln (d_2/r_o - 1) \tag{6}$$

The displacements of the other fibers can be obtained via

$$w_i = \frac{F'}{2\pi G} \ln (r_1/r_2), \; i \neq a \text{ or } c \tag{7}$$

where the field point is the center of the fiber whose displacement is being calculated. Thus

$$w_b = \frac{F'}{2\pi G}\sqrt{1+1/\alpha^2}$$

$$= w_a \frac{\ln \sqrt{1+1/\alpha^2}}{\ln (d_2/r_o - 1)} \tag{8}$$

$$w_d = w_b \text{ (by symmetry)} \tag{9}$$

and

$$w_e = w_a \frac{\ln 2}{\ln (d_2/r_o - 1)} \tag{10}$$

Thus, (5a) can be written as

$$F'_a = G[\left(\frac{h}{d}\right)_{c,v} \left(1+1/2 - \frac{\ln\sqrt{2}}{\ln(d_2/r_o-1)}\right) +$$

$$\left(\frac{h}{d}\right)_{c,h} \left(1 - \frac{\ln \sqrt{1+1/\alpha^2}}{\ln (d_2/r_o-1)}\right)] \; (2w_a) \tag{11a}$$

$$= G\left(\frac{h}{d}\right)_{i,v} (w_a - w_c) \tag{11b}$$

Employing (4) for $\frac{h}{d}$ for a pair of isolated fibers, $\left(\frac{h}{d}\right)_{i,v}$ can be replaced by

$$\frac{\pi}{\ln[(1\pm\sqrt{1-(2r_o/d_2)^2})/(2r_o/d_2)]} \tag{12}$$

Again by noting that $w_a = -w_e$, the above equality becomes

$$\left(\frac{h}{d}\right)_{c,v} \left(1.5 - \frac{\ln\sqrt{2}}{\ln(d_2/r_o-1)}\right) +$$

$$\left(\frac{h}{d}\right)_{c,h} \left(1 - \frac{\ln \sqrt{1+1/\alpha^2}}{\ln(d_2/r_o-1)}\right)$$

$$= \frac{\pi}{\ln[(1\pm\sqrt{1-(2r_o/d_2)^2})/(2r_o/d_2)]} \tag{13}$$

The above equation is a linear algebraic equation in two unknowns $(h/d)_{c,v}$ and $(h/d)_{c,h}$. A second equation can be found by applying a similar approach to Case B.

For this case we obtain

$$\left(\frac{h}{d}\right)_{c,v} \left(1 - \frac{\ln\sqrt{1+\alpha^2}}{\ln(d_1/r_o-1)}\right) +$$

$$\left(\frac{h}{d}\right)_{c,h} \left(1.5 - \frac{\ln\sqrt{2}}{\ln(d_1/r_o-1)}\right)$$

$$= \frac{\pi}{\ln[(1 \pm \sqrt{1-(2r_o/d_1)^2})/(2r_o/d_1)]} \tag{14}$$

Solving (13) and (14) simultaneously, we obtain

$$\left(\frac{h}{d}\right)_{c,h} = \frac{A - (B/F)\,C}{D - C(E/F)} \tag{15}$$

and

$$\left(\frac{h}{d}\right)_{c,v} = \frac{B - \left(\frac{h}{d}\right)_{c,h}(E)}{F} \tag{16}$$

where

$$A = \frac{\pi}{\ln[(1 \pm \sqrt{1 -(2r_o/d_1)^2})/(2r_o/d_1)]}$$

$$b = \frac{\pi}{\ln[(1 \pm \sqrt{1 -(2\alpha r_o/d_1)^2})/(2\alpha r_o/d_1)]}$$

$$C = 1 - \frac{\ln \sqrt{1+\alpha^2}}{\ln(d_1/r_o-1)}$$

$$D = 1.5 - \frac{\ln\sqrt{2}}{\ln(d_1/r_o-1)}$$

$$E = 1 - \frac{\ln\sqrt{1+1/\alpha^2}}{\ln(d_1/\alpha r_o-1)}$$

$$F = 1.5 - \frac{\ln\sqrt{2}}{\ln(d_1/\alpha r_o-1)}$$

Theory Underlying Electric Analogue

If a set of n infinitely long rigid rods immersed in an infinite elastic matrix are displaced axially from their equilibrium positions by distances $w_1, w_2 \ldots w_n$, the distorted shape of every cross-section will be the same. Therefore there will be no direct stress in the matrix, and the only shear stresses will be $\tau_{xz} = \tau_{zx}$ and $\tau_{yz} = \tau_{yz}$. At the interface between the i'th rod and the matrix the equilibrium equation is

$$\frac{dF_i}{dz} = \oint_i (\tau_{zx}\, dy + \tau_{zy}\, dx) \tag{17}$$

where $\oint$ signifies integration around the circumference of the i'th rod. In the matrix, equilibrium in the z-direction requires that

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = 0 \qquad (18)$$

Since the only displacement is $w(x,y)$, application of Hooke's law leads to

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 \qquad (19)$$

in the matrix, and

$$\frac{dF_i}{dz} = G \oint_i \left(\frac{\partial w}{\partial y} dx + \frac{\partial w}{\partial x} dy\right) \qquad (20)$$

at the interface with the i'th fiber.

Next consider a geometrically similar array of n perfectly conducting rods immersed in an electrolyte of resistivity ρ_e. Let the potentials applied to the rods be $\phi_1, \phi_2 \ldots \phi_n$. The current flowing from the electrolyte into a unit length of the i'th rod is then

$$\oint_i (j_y\, dx + j_z\, dy) = \frac{dI_i}{dz} \qquad (21)$$

In the electrolyte Ohm's law leads to

$$\rho_e \vec{j} = \text{grad}\ \phi \qquad (22)$$

and for steady currents

$$\nabla \cdot \vec{j} = 0 \qquad (23)$$

Thus in the electrolyte

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (24)$$

and at the periphery of a rod

$$\frac{dI_i}{dz} = \frac{1}{\rho_e} \oint_i \left(\frac{\partial \phi}{\partial y} dx + \frac{\partial \phi}{\partial x} dy\right) \qquad (25)$$

Comparing (19) and (20) with (24) and (25) we see that ϕ is analogous to w, I is analogous to F, and ρ_e is analogous to 1/G.

The analogy provides a convenient means for an experimental determination of the interaction between fibers. If we assume the validity of (2), we can write

$$\frac{dI_{ij}}{dz} = \frac{h}{\rho_e d}[-\phi_{i,j-1} - \phi_{i-1,j} + 4\phi_{i,j} - \phi_{i+1,j} - \phi_{i,j+1}] \qquad (26)$$

If we measure the total current per unit length in the i'th rod when known potentials are applied to that rod and its four nearest neighbors, we can use (26) to determine $h/\rho_e d$. By measuring ρ_e we can find h/d for a square array. This was done in (4).

The case of a rectangular array is a bit more complicated. The square array leads to (26), a linear algebraic equation in one unknown, h/d. As noted in the preceding section, in a rectangular array there are two unknowns, h/d for the two nearest neighbors and h/d for the next-to-nearest neighbors. Thus two experiments must be carried out, leading to two simultaneous equations in two unknowns.

<u>Experimental Setup</u>

The structure of the composite was simulated in the electric analogue by conducting rods replacing the fibers and a weak electrolyte replacing the matrix. A transparent plastic tank was used to contain the fluid and the rods.

The experiment was carried out first by inserting one end of the rods into a plexiglass panel with holes that match the size of the rods and with spacing that meets the r_o/d and α under investigation. This set up was then loaded into the electrolyte. Alignment of the electrodes was provided by inserting the other ends into a similar panel that can be mounted at any convenient level above the electrolyte.

Two experiments were performed for each α and each r_o/d. The first one involved the application of a constant AC potential across rods a and b as in Figure 4. The two outermost rods were connected and thus d was at the same potential as b, while c and e were left free to adopt the local potential of the electrolyte.

The governing equation can be written as

$$\frac{dI}{dz} = \frac{1}{\rho_e}\left[\left(\frac{h}{d}\right)_{c,v}(\phi_{ac}+\phi_{ae}) + \left(\frac{h}{d}\right)_{c,h}(\phi_{ab}+\phi_{ad})\right] \qquad (27)$$

I was obtained by measuring the current generated by the power supply. z was the submerged length of the rods. ϕ_{ac} and ϕ_{ab} were obtained by measuring the potential difference between a and c, and a and b, respectively. By symmetry, $\phi_{ae} = \phi_{ac}$.

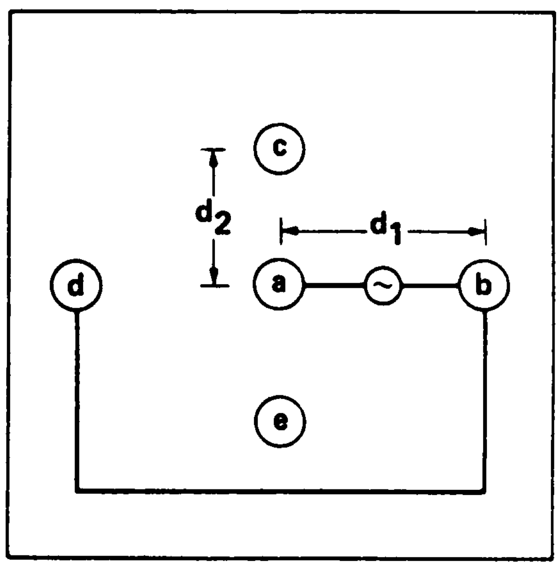

FIG. 4 POTENTIALS APPLIED TO NEXT-TO-NEAREST NEIGHBOR PAIR

In order to obtain a second equation relating the two unknowns, $\left(\frac{h}{d}\right)_{c,v}$ and $\left(\frac{h}{d}\right)_{c,h}$, a constant potential was next applied across rods a and e with e connected to c as in Figure 5. The preceding governing equation again applies. Similarly, I, ϕ_{ae} and ϕ_{ab} were measured for this case. By solving the two linear algebraic equations simultaneously, values for $\left(\frac{h}{d}\right)_{c,v}$ and $\left(\frac{h}{d}\right)_{c,h}$ were obtained.

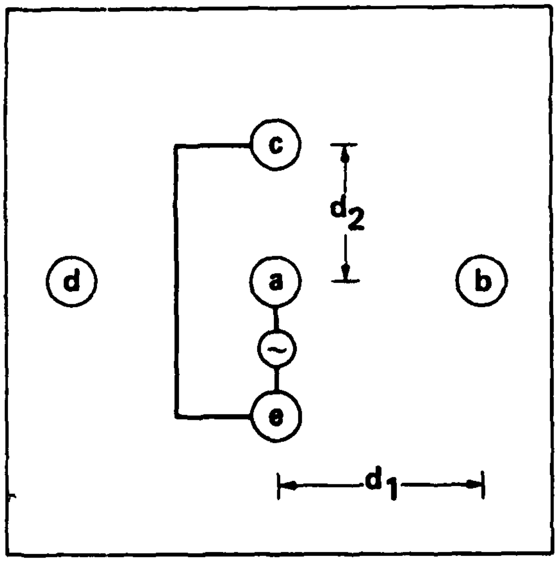

FIG 5 POTENTIALS APPLIED TO NEAREST NEIGHBOR PAIR

Results

The results are summarized in Figures 6-11.

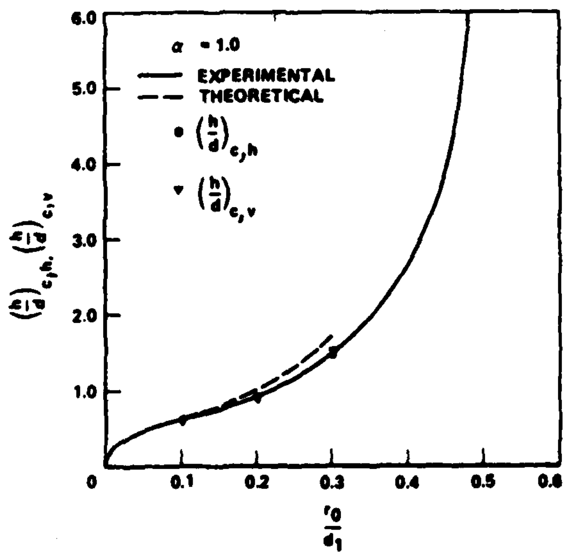

FIG. 6 h/d FOR SQUARE ARRAY

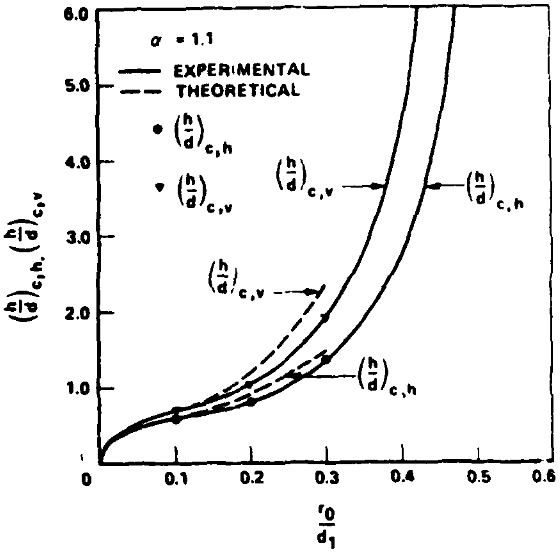

FIG. 7 h/d FOR RECTANGULAR ARRAY

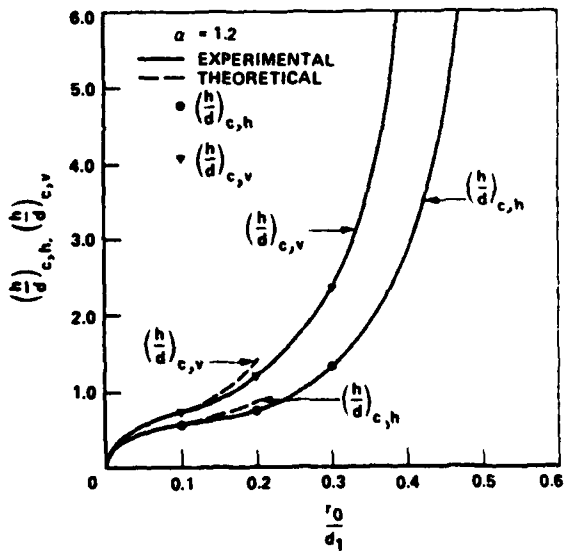

FIG. 8 h/d FOR RECTANGULAR ARRAY

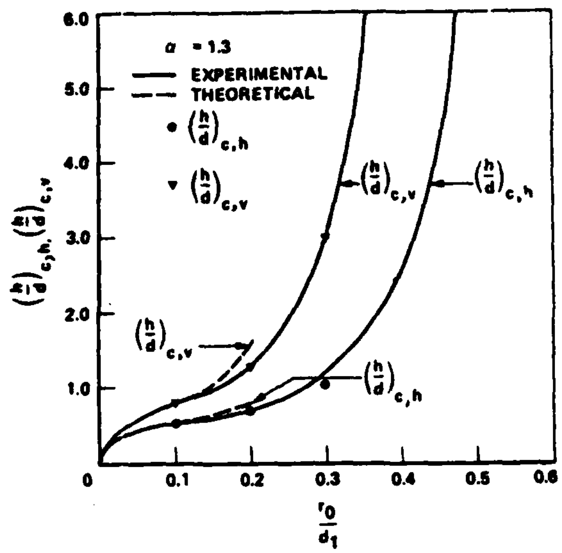

FIG. 9 h/d FOR RECTANGULAR ARRAY

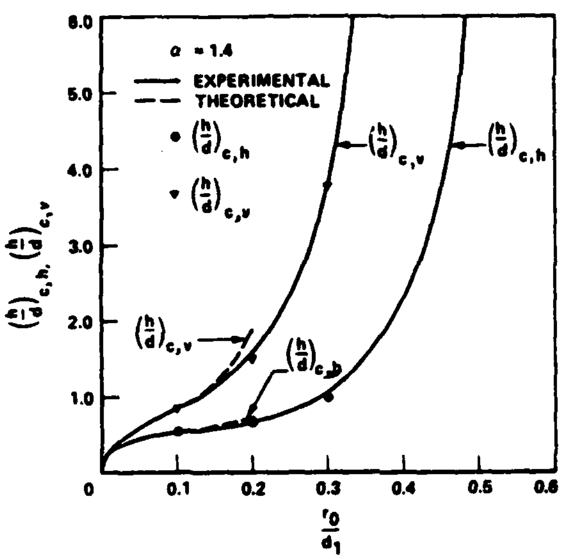

FIG. 10 h/d FOR RECTANGULAR ARRAY

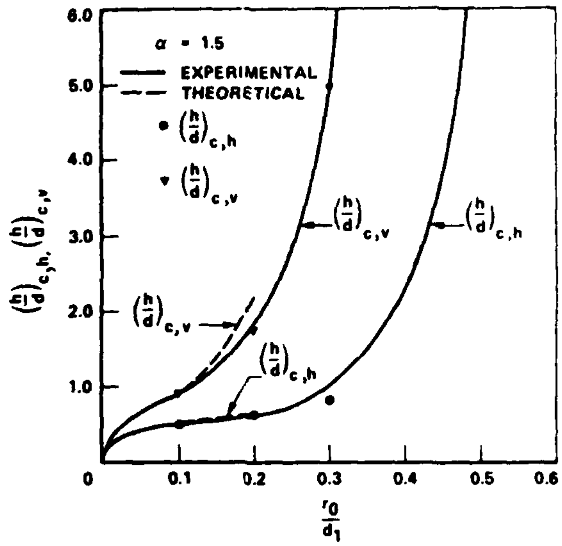

FIG. 11 h/d FOR RECTANGULAR ARRAY

Figure 6 gives h/d as a function of r_0/d_1 for a square array, i.e., $\alpha = 1$. This case was treated earlier in Reference 4. The other figures treat the cases $\alpha = 1.1$ to $\alpha = 1.5$. For $\alpha > 1$ the array is rectangular and therefore there are two curves, one representing (h/d) for the vertical interaction and the other for the horizontal interaction. In each case d in the abscissa was chosen to be the longer distance d_1; this was of course an arbitrary decision. In the case of the ordinates d is d_1 for the horizontal interaction and d_2 for the vertical interaction.

In every case h/d approaches zero as r_0/d_1 approaches zero, and in this region the theoretical solution is valid. It is therefore used to estimate the value of h/d at such small values of r_0/d_1 as to make an experimental determination difficult.

As $r_0/d_1 \rightarrow 0.5$, the horizontal separation between two fibers vanishes and $(h/d)_{c,h} \rightarrow \infty$. This asymptote can be used to extrapolate the experimental data to higher values of r_0/d_1 than can readily be investigated experimentally. As $r_0/d_2 \rightarrow 0.5$ the value of $(h/d)_{c,v} \rightarrow \infty$ because then the vertical separation shrinks to zero. In this case $r_0/d_1 = r_0/\alpha d_2 = 0.5/\alpha$, which serves similarly as an asymptote.

Discussion

In Hedgepeth's papers (1,2) no attempt was made to evaluate h/d for a particular composite, and no technique was proposed for doing so. These papers evaluated the stress concentration factors in fibers adjacent to a crack, and it turns out that the stress concentration factor does not depend on the value of h/d. However to find the stresses at which crack extension occurs it is necessary to find lengths of the overloaded fiber elements, and these do depend on h/d. In the absence of a theoretical technique for accomplishing this, various assumptions have been made. The most common has been to

assume that for the vertical interaction (interaction between closest neighbors) $d = d_2$ and $h = d_1$, while for the horizontal interaction $d = d_1$ and $h = d_2$, Refs. (6-8). These assumptions lead to the results

$$(h/d)_{c,v} = \alpha \tag{28a}$$

$$(h/d)_{c,h} = 1/\alpha \tag{28b}$$

a result independent of r_o/d. Comparing these results with Figures 6-11 we see that they are somewhere near right for $r_o/d_1 = 0.2$, but are too high for smaller values of r_o/d_1 and too low for higher values of r_o/d_1. We note that when r_o/d_1 is 0.2 the fiber volume ratio is about $\alpha/8$, a lower value than what is usually encountered in practical applications. For a square array having a fiber volume ratio 0.5, $r_o/d \approx 0.4$, in which case the value we obtain (see Figure 6) is about 2.5 times that given by (28).

Concluding Remarks

During the last three decades shear lag has been widely used as a technique for finding the stresses in damaged uniaxially reinforced composites. The applicable equations contain a parameter representing the shear interaction between neighboring fibers. Somewhat surprisingly, up to now little has been known about how to evaluate this term for a particular composite.

It is shown that for small values of the fiber diameter to fiber separation this parameter can be evaluated theoretically. For larger values it can be evaluated experimentally employing an electric analogue. By combining the two techniques, the parameter is evaluated over the entire range of diameter to separation ratio for composites in which the fibers form a rectangular array. The results contained in Figures 6-11 cover arrays ranging from square to rectangles having a side ratio of 1.5.

Acknowledgement

This research was sponsored by the Office of Naval Research under Grant N00014-76-C-0445. Dr. Y. Rajapakse was the technical monitor.

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